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History of Applied Geometry

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HISTORY
OF
APPLIED GEOMETRY

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INTRODUCTION

Mathematics! Just what does the word mean to us? After a moment of thought many different meanings may present themselves to our minds. At first we are inclined to say that the word mathematics covers a vast field. We are justified in so thinking because mathematics embraces a wide scope of study. Were we to say that it is a science we should place it in its proper genus, for it is truly a science of numbers and space. However, could not the science be the art of calculation or the art of computation? Descartes asserts that, "All which have for their object the search after order and measure, belongs to mathematics." He makes his assertion from the fact that mathematics is based on the concept of magnitude, order and measurement. After deep study we conclude that mathematics is a series or combination of sciences that treats of the relation between qualities using numbers and symbols in the operations which demonstrate those relation; the same is to be applied by the reasoning power of men to the solution of all numerical problems of business affairs and of practical life.

Knowing the operations and demonstration of mathematics and knowing how to apply them are two vastly different accomplishments. Anyone who has studied mathematics in its elementary or advanced phases know its theories and its relations. But how many give any particular thought to the practical application of mathematics? I am presenting this question to the average college student. When we look back the days of

our elementary education, the value and usefulness of arithmetic is evident. We learned addition, substraction, multiplication, division, and other operations. These functional skills and operations are so valuable and so useful that we apply them in practical life without conscious consideration of their necessity.

In high school, we learned algebra which was a new and fasinating study almost monopolizing our interest. We found that by applying its operations we had a quicker and easier understanding of practical problems that had been worked before in arithmetic. Unfortunately the average student, when coming in contact with higher branches of mathematics such as geometry, trigonometry and calculus, and finding no spontaneous practical use for them loses interest. In fact, it is at this stage that the majority lose sight of the application of these branches of mathematics and become interested in the theories and operations of the function only. We very seldom or never think of these branches as having practical application, and we rarely interest ourselves enough to find out how they may be put to practical use.

It is now my aim and purpose to assert clearly and to prove that geometry from the very beginning until this present time is a branch of mathematics that has always had and will always have practical applications. The very history of geometry shows emphatically and decisively that it is a science of various applications. In order to prove this I will embrace the history of geometry stressing in practical phases

and giving scant attention to theoretical or classical geometry.

EGYPTIAN INFLUENCE

Let us begin with the origin of geometry. The word geometry is derived from two Greek words meaning "earth" and "I measure" and was onced called surveying. It is supposed to have had its origin in land surveying, although it is comparatively easy to distinguish between the abstract reasoning of geometry and its present day practical rules of the land surveyor. Universal tradition is that geometry originated in Egypt. As has been said before geometry is derived from two Greek words meaning "earth" and "I measure." It follows then that geometry grew out of the necessity of surveying.

All early geometry was intuitive in its nature. It sought facts relating to mensuration without attempting to demonstrate these facts by any process of deductive reasoning. The prehistoric geometry sought merely agreeable forms as in plaiting of symmetrical figures in mats.

In the seventh century we hear of the Egyptian using geometry, although these people carried this science no further than was absolutely necessary for their practical use. The Egyptian first had occasion to use geometry in laying out the ground plan of certain buildings. They also found the exact position of the sacrifical altars by means of practical geometry. The geometric method used by the Egyptians to get a right angle which was by means of a rope divided by knots at "B" and "C" so that the lenght AB, BC, and CD were in the ratio

of 3:4:5, is at this present time used by practical engineers in the measuring of a right angle.

In the cultured and civilized age of Egypt geometry played an important part in the measuring and leveling of land. The early surveyors had no telescope or microscope to measure distances. A rope or a wooden rod was used, the unit of measure varying in different localities. They laid off right angles by means of a cross placed horizontally on a staff resembling the carpenters square of the present time.

Menes, the first King of Egypt, built pyramids, the first temple of Phthah at Memphis, and changed the course of the Nile River. It is quite evident then that a people engaged in such gigantic enterprise must have known something of practical geometry.

Other Egyptian kings used geometric methods to divide the land equally among their subjects so that they might impose a yearly tax for revenue.

In the year 3000 B.C. Egypt was in a period of rapid development in practical engineering. There is no authentic document giving direct information but the achievements of the engineers such as the building of the Great Pyramid of Gizith gives sufficient proof that geometry was used extensively.

The Egyptian engineers learned very early to carry a straight line over elevations of the earth's surface, or a plane around the bend of the Nile. The degree of accuracy of these surveyors was such that later critics say the maximun error in measurement was a little more than one-half inch.

All specimens of Egyptian geometry deal with practical numerical problems and not with general theories. Whenever a result was stated as a universal truth, it was only declared a truth after a series of inductive processes.

Directions for obtaining the "Knowledge of all Dark Things" a papyrus, the most ancient, which is now in the British Museum puts us in contact with Egyptian geometry. We find no mention of the theoretical results. There are very few general rules or procedures but mainly statements of results derived from practical problems, probably to be explained to pupils by the teacher. The chief ideas of the Egyptians in geometry were making constructions and determining areas. A great many of their rules, especially those in solid geometry had probably not been proved at all but were known to be true from observation or accepted without proof.

GEOMETRY UNDER GREEK INFLUENCE

In the first chapter we found that the Egyptians used geometry only for its practical value but here we will find that from the moment the Greecian philosophers applied themselves to the study of Egyptian geometry the science took on a radically different aspect. The Greeks developed geometry from a mere practical to a theoretical or classical study.

Geometry was used in the time of Plato and Aristotle and goes back as far as Thales. In fact, it was used from the time of Thales until the decay of their ancient civilization, then on during the middle ages it ceased to developed but was used in architecture. It began to flourish again in the six-

teenth century and on until the present day.

Geometry as a logical science is purely a product of Greek and other western civilizations. The Greeks formulated into scientific language the truths which the Egyptians saw and merely felt to be true. The Egyptians studied solid geometry and earth geometry while Thales in Greece developed geometry of lines.

Greek geometry may be divided into two schools. The classic or theoretical school, in which the philosophers and teachers knew that geometry was a science that could be applied to practical problems but thought it undesirable to seek its applications. It is to this school that Plato and Euclid belong. The other school may be spoken of as the school of theoretical and applied sciences. They knew that geometry could be applied and set about to make practical use of it. It will benefit our purpose to treat of the school of theoretical and applied science. To this school belong Thales, Pythagoras and Heron the Elder.

However, the first geometry used in Greece was connected with art. The Greecians appreciated geometric forms just as the Egyptians, therefore they based their art on geometry. This is first seen in crude parallels, then in the more elaborate forms in the advanced ages to the more delicate forms of the higher types of Greek art.

The honor of having introduced the study of geometry as a science into Greece belongs to Thales, one of the seven wise men, a man distinguished for his great mathematical ability,

his shrewdness in business, and his fame in science. He studied mathematics and the physical sciences with the Egyptian priests. He is said to have excelled his master in measuring the height of a pyramid from its shadow. This Greek greatly improved and developed geometry. Propositions concerned with the measurement of an angle and a straight line are attributed to him. Thales invented the theorems on bisection of a circle; the equality of vertical angles; and the congruence of two triangles having a side and two adjacent angles equal. Thales was the first Greek to apply theoretical geometry to practical use by taking the theorem on the congruence of two triangle to measure the distance of a ship from the shore. Thales also introduced Astronomy into Greece.

Pythagoras, a contemporary and student of Thales, studied in Egypt where he imbibed all of his scientific knowledge. Geometry and arithmetic were his favorite subjects. Like Egyptian geometry, the geometry of Pythagoras was much concerned with practical problems. On his return to Greece he attempted to settle in Samas and found a school, but he found this in such a socially upheaved condition that educational progress was impossible. Failing to settle there he journeyed to Craton and founded the famous Pythagorian school. which progressed rapidly and gained much recognition. Its imitation of Egyptian usages and its aristocratic tendencies caused it to become an object of suspicion. Because of this the school was destroyed and Pythagoras was forced to flee to Tarentum where he was murdered.

Pythagoras taught and introduced many geometric theorems but very little of his work was saved from destruction of his school. He and his pupils applied their knowledge of geometry to mechanics. They are supposed to have invented and worked out the theory of the pulley. They are also credited with the construction of a flying bird and other mechanical toys.

The last of the Greecian school of theoretical and applied science whom we shall consider is Heron the Elder of Alexandria celebrated for his clever mechanical inventions. He wrote "Treatise on the Dioptra" which contains solutions of a large number of geometric questions, such as finding the distance between two points of which only one is attainable; and to measure the area of a field without entering it.

Heron was a practical surveyor, he placed surveying and engineering on a scientific base with the aid of practical geometry. He did not work with theoretical or classical geometry but discussed only applied mathematics. His works consist of applications of geometry to determine the area of fields of different shapes; propositions on finding volumes of certain solids with applications to theaters, baths, banquet halls etc; rules to find the height of an unacessible object; and a table of weights and measures. In applied mathematics Heron invented one hundred or more mechanical toys, a double force pump to be used for fire engine; and a small stationary stream engine.

Heron's geometry is very different from classical geometry. He did nothing to extend the theoretical knowledge of mathematics. He was interested in science and he greatly desired to bring out its practical applications. His writings satisfied practical wants. Traces of them were found in Rome and in the Occident during the middle ages.

GEOMETRY IN THE MIDDLE AGES

From about 500 B.C. through the middle ages the development of geometry as a science took a steady decline. In fact its progress on a whole stopped. What little attention was directed to mathematics was applied to arithmetic and algebra. Thus we must leave geometry during this period and consider it again in the Modern period of Mathematics.

However, during the period of the middle ages Boethus, the founder of scholastic philosophy contributed a geometry consisting of only a selected number of propositions, but these few propositions had numerous practical applications in finding areas.

The first Roman mathematician of any note during this era is Isidorus, who produced works in arithmetic, music, geometry and astronomy. He applied his geometry in most of his astronomical works.

The Arabs also appreciated geometry and its applications to astronomy. Although very little attention was given directly to geometry as a science, there are many traces of its applications in optics, astronomy, architecture and mosaics.

Some of the theorems and propositions that the Greeks never dreamed would be put to practical use were extensively applied in astronomy and architecture during this period.

The conic section in geometry assisted astronomers to trace the path of the planets in their orbits. The volume of solids were applied by wine merchants.

In architecture the circle and the triangle have been the foundation of magnificent edifices. Various patterns for church windows, arches, and steel ceiling had the triangle and circle as bases for their designs. Architectural ornamentation of leaded glass windows illustrate the use of concentric circle, quadrifoils and like geometric figures. The Gothic Arch was based on the equilateral triangle. Arcs and circles predominated in many artistic designs. Saint Peter's tomb is a wonderful specimen of geometric figures. Mosaic ornamentation which consisted entirely of geometric patterns attained its full development during this period.

In the field of astronomy we found Hipparchus to be the most emiment astronomer of Greece at this time. He observed and calculated the longitude of a certain star to be ninety degrees. His approximate determination of the duration of the year brought him much renown. He worked out the eccentricity of the solar orbit within the calculation of a few minutes. The inclination of the equator was also accurately calculated by him. Hipparchus was interested in mathematics only so far as it aided him on his astronomical observations. If he had had no knowledge of geometry and its angular meas-

urements he would not have been able to make the interesting discoveries which are attributed to him.

Potolemy an Egyptian by birth is another in the field of astronomy, who preserved his name in history by his works concerning the planetary system. A great many of his works were based partly on his own researches and partly on those of Hipparchus. Potolemy introduced the theory that the sun and planets revolve around the earth which is the center of the system. Although we all know that this theory is wrong still, we can see that he had enough mathematical knowledge to make astronomical observations and come to what he and some of his contemporaries thought to be the right conclusion. He wrote a treatise consisting of directions for determining the position of the sun and the rising and setting of the stars.

Although much interest and attention was given to astronomy in the middle ages, this attention was small when compared to the interest and attention given to architecture.

Artistic construction began in the Babylonian and Egyptian age and Greece perfected it with her Ionic and Doric structures. However, we do not see this classic science in full bloom until the middle ages, when beautiful buildings especially cathedrals with large stained glass windows, lofty vaults, slender spires and mosaic ornamentations, stand out in bold relief.

It was during this time that Gothic architecture came into prominence. This form of architecture appreciated and

applied beautiful geometric forms, not merely with respect to decoration but in the general structure of cathedrals.

Architecture never played so important a part at any time as it did in the middle ages. The Gothic structure with its artistic beauty, became the essential style of architecture everywhere. It spread in France, in Spain, in England and in Germany. Cathedrals were built in every small town, in fact, cathedral building had almost become a contest. Cathedrals were built with a two fold aim during that time. All attention was not focused on beauty but the greatest care was taken that the structure should be durable.

On entering a Gothic church to-day, one is immediately impressed by its magnificent structure, beautiful decoration and exquisite sculpture.

Mosaics were an essential element in the decoration of these cathedrals. This ornamental work attained its full development during the middle ages.

Mosaic art is the grouping into geometrical patterns of pieces of marbles, glass or enamel so as to form an artistic design. It was used on the walls, floors, monuments or furnitures of all cathedrals. During the middle ages all floors were of a simple form of mosaic and for artistic decorations the higher types were applied. The shapes of marble were sometimes three millimeters in size. These patterns were gilted around the edges and colored with metallic oxide. Mosaics were almost always placed in the triumphal arches, apex and sometimes on the walls of the naves of the cathedrals.

All twisted colums, tomb monuments, altar canopies, pulpits, choir scenes and other details of the middle ages were inlaid in geometric design.

The mosaics were generally taken from subjects of the Old and New Testaments. The small cubes were worked in patterns of circular, square, long or polygonal slabs of rich marble. These geometric patterns were transposed into exquisite designs. The geometrical mosaic pattern even worked itself into architecture. It formed the main decoration for porticoes, and even cloisters.

Although theoretical geometry was not developed and advanced, we see that it was more practically applied than ever before.

With these remarks let us leave the geometry of the middle ages and enter that of modern times.

PERIOD OF MODERN GEOMETRY

The Modern Period of Mathematics began in the early part of the fifteenth century. Geometry of this period is very different from that of the earlier periods we have traced. It is far more complex and complicated. Geometry of this period is both analytical and synthetic. Unlike early geometry it was not influenced by any particular nation or group of people but several individuals have contributed to extend geometry both theoretically and practically.

Many are the brilliant and celebrated mathematicians of the modern period, who have been deeply interested in geometry and its applications have been numerous. We have noticed that

est concern. Much of his time was devoted to optics in determining the shape of lenses of telescopes. This has done much for the advancement of our astronomical science of today. Due to the powerful lens on our telescopic devices men can study more thoroughly the mysteries of the heavens. Even in our own day we have a discovery recorded that is indirectly due to Descartes, e.g. the discovery of the new planet "X". It is by the aid of the telescope that our astronomers can explain atmospheric phenomena, such as the rainbow and its courses. So too, do we have some notion of the wisdom of that Power behind the never conflicting motion of the numerous heavenly bodies.

Descartes discussed the motion of the universe and laid down the laws of its nature. He was a mathematical genius but he had an extended imagination which sometimes went far beyond his own concepts.

Desargue is a contributor to the seventeenth century geometry. Very little is known of his early and personal life. However, it is known that he was an officer in the army and later became a practical engineer. He had a great knowledge of mathematics, geometry in particular. The interest in mathematics shown by Desargue was in its application to the study of perspectives which he introduced. This art of perspective is extensively used in painting, engineering and architecture.

Neuton is another important spoke in the wheel of geometry. Although, Neuton as a child was a very sickly and in-

attentive student, carpentering mechanics, and drawing seemed to be his main interests. He achieved very little until after he had received his master's degree where his genius asserted itself and he was considered the most prominent mathematician of the seventeenth century. He was appointed later professor of mathematics at Cambridge.

Neuton's idea was to apply mathematics to the phenomena of nature. These ideas are clearly shown in one of his works known as "Principia". Motion is the subject treated most in his treatises. The central idea was that every particle of matter attracts every other particle in the universe. The entire work is demonstrated by geometrical principles.

He applied his theorems to the phenomena of the solar system and to determining the distance and the mass of the planets. He observed the comets, determined their orbits and showed that they belong to the solar system.

Neuton did not establish any new methods in geometry but he has shown great power and ability in applying it. He established the law of universal gravity and created physical astronomy. He explained geometrical decomposition of light and the theory of the rainbow.

Poncelet, one of the founders of modern geometry was a great lieutenant of engineers. While taking part in a Russian war he was abandoned as dead on the field of battle and was later taken as prisoner. Having been deprived of all books he was forced to rely on his memory for all his knowledge. It was because of this that he became famous by his original

mathematical researches. While in prison Poncelet wrote a book, consisting of the principle of continuity and the properties of projective figures. This book has come down to our own day. His treatises have rendered great and excellent service in various practical applications. Much of his time was given to mechanics and to hydraulics.

Simon Newcomb of the twentieth century was a noted and celebrated mathematician and astronomer. He was essentially self-taught although he attended the scientific school at Howard University. His main research works were in astronomy. He gave a distorted description of the motion of all the planets. Methods for accurately computing the orbits of planets, asteroids and comets which are used by astronomers to-day were formulated by him.

It is men like these who have made possible the wonderful inventions and discoveries of this modern age. Their mathematics contributions are the links in the scientific chain of to-day.

In this age of rapid development of huge and complex construction, the engineer, the architect, the carpenter, and the builder in general needs a full knowledge of geometry. The architecture or engineer for example, who has no knowledge of the principles and the methods of geometry along with his knowledge of masonary and electricity and his ability to make intelligent estimations would be as much out of place as an instructor with no knowledge of his subject.

In all problems of construction geometry is necessary. The application of parallel lines and correct angles are absolutely essential in the erection of any structure.

Triangles play an important part in the study of geometry and also in its application to arts and industries, especially in engineering. Before erecting a structure of any kind the engineer must make many measurements and draw many maps and plans. Angles of depression and elevation, horizontal angles and lines must be measured. In the building of steel bridges, triangles and arcs are used to give strength and rigidity.

Engineering is designing and constructing. If there were no geometry would there be any engineering? This science has played an important role in the life of man. Engineering does not only consist of designing and constructing but often operates the various structure and machines that serve as ways and means of communication of securing and utilizing the natural store of wealth and of protecting life and property from the actions of the elements. To these ends roads, railways, bridges, canals, harbors, docks, mines and factories are constructed. These work shop and factories are equipped with the highest obtainable grade of mechanical powers. Most of the arts and sciences are called into use, and geometry plays an important part.

In this rapidly increasing activity engineering is indispensable especially, when to-day's aim is to secure a maximum expenditure.

In this age of careful provision for human comfort there is a demand for public water supplies, sewerage and drainage, fire protection, the provision for light, heat, and power.

There is also military engineering. Officers of the army and navy have designed and constructed fire arms and camps. Offensive and defensive military works are operated by land and sea.

In architecture, in carpentry, in making measurements for any construction many geometric principles and geometric figures are employed.

The carpenter whose main work is to build and decorate houses and other structures specially contributes to the general welfare of the community. Besides a knowledge of the characteristics of his material, he must also have a fair knowledge of geometry and its applications, for geometry is essential in the planning and laying out of the structure.

In laying out the plans for a building a little thought and skill is required. In order to know the best way in which to approach the problem a considerable knowledge of geometry must be had. Suppose we were building a house or any large edifice, the first thing that must necessarily be done is to make a plan. After the plans are made and the ground selected, the next important step is locating the position of the corners of the building. There must be such accurate measuring of angles that in cases of large buildings engineers are selected to do the work. If a mistake is made in the staking off of a building endless trouble is caused

in the erection.

After the building has been laid off the next essential part is its frame or skelton over which an ornamental covering is to be placed.

In these steps of constructing a house we can clearly see that geometric principles such as parallel, horizontal, vertical and intersecting lines; slopes, inclinations and vertical distances; and all types of angular measuremants are applied.

CONCLUSION

From the history of geometry we found that it has its origin in practical problems. In its own way it is as necessary to man as the simple functions of arithmetics. Whenever, we had occasion to think of the applications of geometry, it has always been in some distant field far beyond our reach. We should not consider geometry in this light because it shows itself in our every day life. Take for example, the athletic world.

Geometry plays an important part in the field of athletics. To play football there must be a gridiron; to play tennis there must be a tennis court. Geometry principles are exclusively used in marking off a football gridiron, parallel lines are used in laying out the entire field. It is bound by parallel lines and marked at intervals by the same kind of lines up to the parallel goal lines which intersect at right angles.

The same parallel lines are used in laying off a tennis court or a baseball diamond.

Thus we can see plainly that there is use for geometry, not only in scientific fields but also in the familar grounds of our every day life.